CHAPTER 4

CAMERA CALIBRATION METHODS

Executive summary

This chapter describes the major techniques for calibrating cameras that have been used over the past fifty years. With every successful method there was usually a significant driver such as higher resolution film, post second world war mapping requirements, cheap and powerful computing, electronic sensors, industrial measurement needs.

The methods discussed range from highly expensive multi-collimators that were of national importance to arrays of straight lines formed by buildings, railway tracks, or stretched string. All of the methods had merit in their time and knowledge of these methods can be of benefit in solving new calibration problems as they arise.

4.1 Introduction

With a perfect lens system, light rays would pass from object space to image space and form a sharp image on the plane of focus according to the fundamental physical laws of optics. This model is often referred to as the pin-hole model where no distortion in the images is present and image quality is ignored. The reality of most lenses is that compromises necessary to minimise aberrations and imperfect construction means that elementary formulae are only good as a first approximation. Deviations from theoretically exact models, must be considered and mathematically modeled. Chapter 2 has provided a series of models for lens calibration and this chapter details various ways model parameters can be estimated.

Methods used for the calibration of close range cameras have evolved over the last few decades from those used for aerial cameras, where the application was essentially parallel-axis stereoscopic photography, to techniques which use the geometric conditions of convergent camera views to extract the interior orientation and lens distortion parameters.

4.2 Calibration of lenses at infinity focus

Aerial camera calibration techniques traditionally used optical calibration methods. These techniques were developed from the turn of the century by national map-making organisations as they had ready access to precision theodolites. The concept of observing an angle (direction) from the theodolite through the lens of the camera and onto the image plane was a task requiring little calculation and exploited the skills of their surveyors.

From this basic concept of measuring the angles either side of the camera lens to determine the amount of deviation at the lens, two types of instruments for the purpose of camera calibration were developed: multi-collimators and the goniometer. These devices served map-making well until the late 1960s or early 1970s when lens distortion modeling became scientifically established and the need for more accuracy caused these 'dinosaurs' of mechanical/optical ingenuity to be retired in favour of computerised solutions.

It is worth noting that due to the fact that film resolution was relatively poor until the late 1950's to 1960's decentring distortion was not a significant concern although its effect (noted as the prism effect) was known about. The major concern was to measure radial lens distortion and to achieve this most methods used either a plane array of collimators or a single axis of rotation telescope and required the lens to be rotated through 90 degrees. Hence, lenses were calibrated by measuring the distortion along only two perpendicular axes.

4.2.1. Goniometer

The goniometer technique involved placing a precise grid (often referred to as a reseau plate) on the image plane of the camera and illuminating it from behind so that the images of the grid crosses were projected out into object space. Lenses were generally calibrated at infinity focus using a collimator rotated about the front node of the lens. The principle of autocollimation was used for location of the principal point. Hallert (1960) described the goniometer principle. A precision grid is used with lines in a 10 mm spaced regular array. The grid was illuminated and its etched pattern projected through the lens. The illumination was normally monochromatic. A telescope, focussed to infinity, was directed towards the camera lens. The grid was projected on the collimating mark of the telescope and adjusted into coincidence there. By pivoting the telescope according to Figure 4.1 the angles were measured. By recording the angles to selected intersection points and knowing the grid spacing, it was possible to estimate all of the camera interior orientation parameters.

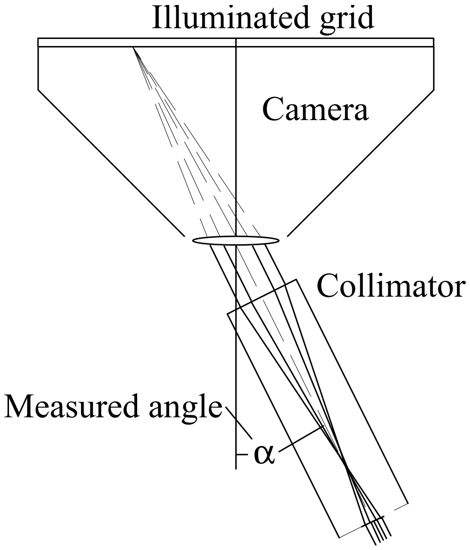

Figure 4.1. The moving collimator goniometer principle

Many goniometers required the lens to be mounted with the principal axis horizontal and rotation of the camera to provide the desired two axes of rotation. An alternative goniometer configuration was similar to a theodolite in that a vertical and horizontal axis were used to measure angles about the point where two mutually perpendicular axes cross (Figure 4.2).

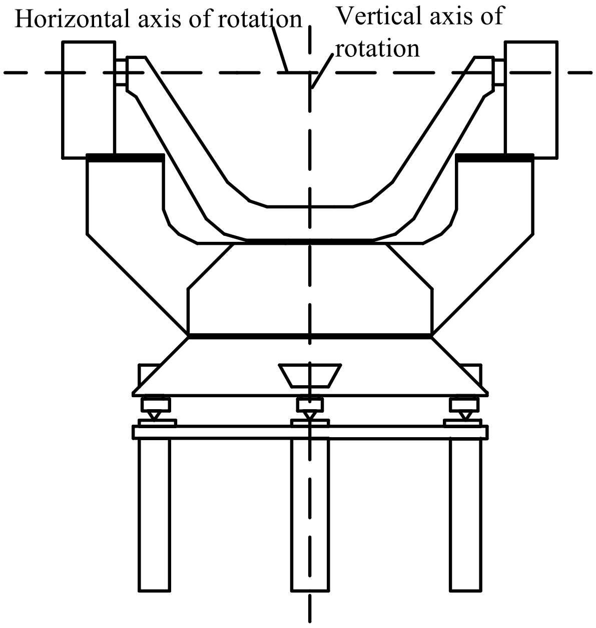

Figure 4.2. The Hilger and Watts Vertical Goniometer

In this system a series of mirrors was used to allow the user to stand beside the instrument and set the collimator in line with a given cross on the grid mounted in the focal plane of the lens. The angle was then read using another eyepiece.

4.2.2. Muli-collimator arrays

Multi-collimators worked with much the same principle as goniometers, except in a reverse sense. Collimators can be thought of as telescopes with illuminated cross-hairs, focused at infinity and pointing at the lens of the camera from a variety of directions. The series (or bank) of collimators shone their illuminated crosses through the lens and onto the image plane of the camera where they were recorded on film (or more likely a glass plate). The positions of the crosses on the exposed plate were observed and, knowing the positions in object space of all the collimators by precise surveying, calculation of lens distortions could be made in a manner analogous to the goniometer technique. The basic scheme is illustrated in Figure 4.3 where each collimator produces an image at infinity of an illuminated cross-hair on the image plane.

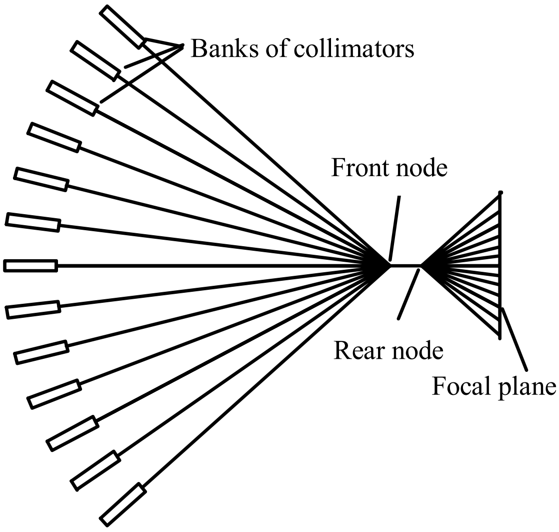

Figure 4.3. Multi-collimator calibration scheme

In Canada, the Canadian National Research Centre initially used a visual method from 1931 but introduced a photographic technique in 1955 as significant differences between visual and photographic methods were found. A second generation photographic method was developed by 1969 which employed collimators to produce 43 targets at infinity with an angular spacing between collimators of 90/32 degrees (Carman & Brown, 1978; Carman, 1969). Off-axis parabolic mirrors were used to eliminate chromatic aberration. The direction defined by each collimator (aperture 63 mm diameter) could be considered independent of the section of the lens (defined by the camera aperture) through which the rays passed. These angles were known to within 0.5 seconds of arc in radial and tangential directions. The photographic plates were measured to a routine accuracy of 1 µm at any field position. The routine calibration accuracy for 99% of measurement amounted to ± 3 µm.

The US Geological survey calibrated lenses from 1953 using a multi-collimator (Karren, 1968). Their system comprised 53 collimators which were mounted in a cross formation. The camera was set up in the following way: the front node of the lens was made to coincide with the point of intersection of the 53 collimators and the focal plane was set perpendicular to the central collimator. Finally the camera was adjusted by tipping such that the plane parallel plate was perpendicular to the axis of the autocollimating telescope (Tayman, 1974).

4.2.3 Stellar calibration method

The angular position of stars is known to a high degree of accuracy and repeatability. Shmid (1974) described the calibration of the Orbigon lens. The standard error in position of the stars was less than 0.4 seconds. Over 2420 star images were visible on each plate. A disadvantage of the method was the requirement to identify each star and apply corrections for atmospheric refraction and diurnal aberration. However, the large number of observations meant that a least squares estimation process was possible. Terms for calibrated focal length, principal point (indicated) and principal point of symmetry, radial and tangential distortion, and orientation of tangential distortion were used. The mean standard error of an observation of unit weight was about 2.7 µm.

4.2.4. Field calibration method

Field calibration makes use of terrestrial features that have been surveyed to relatively high degree of accuracy to calibrate camera lenses. The advantages of the method are: in the accuracy of these points, which have typically been surveyed previously; the fact that the camera can be used in conditions similar to which it will operate; and calibration can take place at a similar time to use. A disadvantage can be the presence (for single camera calibration) or lack (for multi-camera calibration) of 3-D detail. Merrit (1948) describes a rigorous method for "determining the principal distance and the photograph co-ordinates of the plate perpendicular to the field". Other variants of this method have used a tall tower and concentric grids on the ground, and lakes which were considered acceptably flat but still had enough detail for stereo photography (Hothmer, 1958).

4.2.5. Conclusion

The infinity focus methods of camera calibration either required substantial laboratory space or large volumes with known spatial information. All took of the order of a day for a couple of skilled technicians to make the requisite observations. The multi-collimator method was especially demanding of space as the bank of collimators had to be rigidly (often permanently) affixed in blocks of concrete and at such a variety of angles that a laboratory set-up could involve adjoining floors in a dedicated building.

During the calibration calculation phase it was usual to vary the nominal length of the principal distance so as to minimise the radial distortion. An assessment could also be made of the offsets of the principal point, or in fact these values could be varied in order to facilitate symmetry of the lens distortions.

The only method that has found favour with the close range photogrammetric community for calibration at finite distances is the field calibration method. In general, close range applications always have to produce a quick and cost-effective solution to an immediate problem, unlike the situation with aerial work where a camera system may be taken "off-line" for an annual calibration when flying conditions, cloud cover or the like are predicted to be unsuitable.

4.3 Methods of locating the principal point

4.3.1 Autocollimation methods for aerial cameras

Methods of finding the principal point of autocollimation for film cameras at infinity focus fall into two categories. The first (Figure 4.4) is performed with a horizontal camera optical axis and uses two collimators which are in alignment with each other and the replacement of one of the collimators by a mirror surface (Field, 1946).

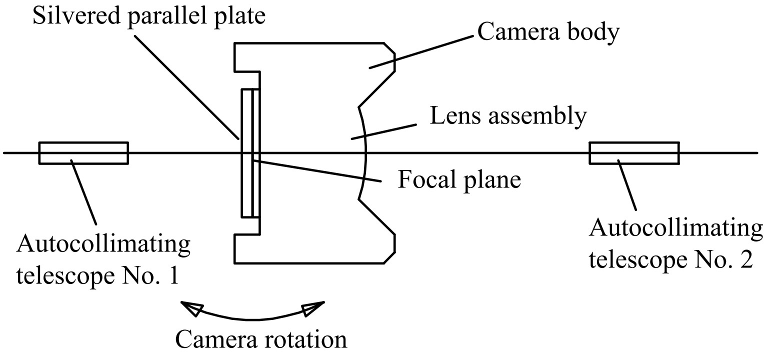

Figure 4.4. Dual collimator method of finding the principal point

The second method is performed with a vertical camera optical axis and uses a mirror surface in place of autocollimating telescope No. 2 in Figure 4.3 (Figure 4.5).

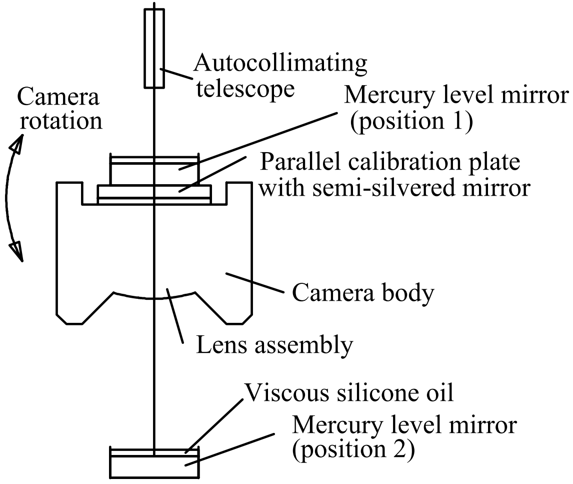

Figure 4.5. Single collimator and a level mercury mirror surface.

These methods, or variants on the same scheme, were used for film camera calibration at infinity focus. The next method can be used with CCD cameras at any focal setting.

4.3.2 Autocollimation method for CCD cameras

A method that can easily be used to find the point of autocollimation uses a laser which is mounted in such a way that it can be adjusted to impinge on the centre of the sensor array and be perpendicular to the image plane. With the laser and the camera in such a configuration the laser beam is attenuated and the lens fitted. Provided that the camera and laser are still in the same configuration, then the location of the focussed parallel beam of the laser is at the point of autocollimation. There are two physical disadvantages of this method: first, the camera cannot easily be calibrated in its working position; second, the location of the point of autocollimation is not guaranteed to remain stable if the lens is knocked, adjusted, or removed and replaced. Hence, useful though this method is, in that it is independent of LSE procedures and avoids correlation with other parameters, it is ultimately not a practical method.

When adjusting the sensor surface to the desired orientation with respect to the laser beam, the reflected beam must be arranged to return along the same path as the incident beam. Care must also be taken not to confuse the returned reflection from any protective cover glass mounted in front of the sensor. The laser beam is coherent and monochromatic and the observed reflections are a diffraction pattern caused by the micro-structure of the silicon surface of the sensor. This diffraction pattern is both regular and symmetric. If the laser if projected through a small hole as shown in Figure 4.6, the returned beam must be aligned to return through this hole. The intensity of the diffraction peaks diminishes from the central peak, which is surprisingly similar to its nearest neighbours, and alignment can be achieved not only by using the central peak but also using the maxima either side of the main peak.

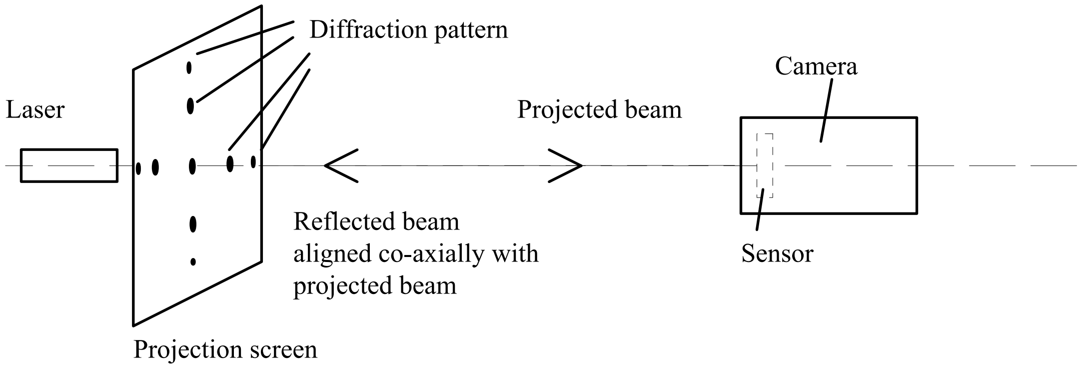

Figure 4.6. Configuration of the laser and sensor

Attenuation of the laser can be achieved by using parallel-sided neutral-density filters, or by use of the exposure control for the CCD camera. In this way the point of autocollimation can be found by locating the image of the laser beam with a typical aperture setting. It should be noted that speckle effects caused by the laser beam will cause the image of the laser beam to be falsely located (Clarke and Katsimbrus, 1994). For this reason it is reasonable to assume accuracy only to the nearest pixel for the point of autocollimation. It is also worth checking that all algorithms use the same image size and investigating the operation of the frame-grabber as the full sensor size is not always digitised.

4.3.3 Determination of the principal point location by analytical techniques

The principal point can also be determined as a parameter in a self calibration technque that will be discussed in the next section.

4.4 Laboratory and test ranges for calibration of cameras at finite focal distances.

The construction in photogrammetric laboratories of elaborate test ranges containing a hundred or more targets, meticulously coordinated to a few tenths of a millimetre by theodolite intersections or determined from precise measurements of images taken with metric cameras, belongs to the era of the late 1950s to early 1980s. This method of calculating camera calibrations was most popular in universities and virtually all departments with an undergraduate or post-graduate reputation in photogrammetry boasted such a facility. Test ranges with coordinated targets photographed from a single point of known position can provide a solution to the problem of camera calibration.

The upkeep of such a test field is not inconsiderable and the results obtained for close-range cameras often approached a root-mean-square residual on the image plane of a few micrometres. Computers and least squares analyses were used to process the result. From the mid-1970s it became widely known that improved results could be obtained by techniques that did not require prior knowledge of the exact coordinates of all the targets on the test range, and this news sounded their death-knell. By combining multiple photographs from convergent angles together in a simultaneous solution, (called the bundle adjustment), it was possible to use the collinearity equations, together with additional parameters, to solve for the camera locations and orientations, the parameters of lens and camera calibration and the coordinates of the targets as well.

Duane Brown pioneered this technique which he termed Simultaneous Multi-frame Analytical Calibration (SMAC) and its only caveats are that film deformation, film flatness and focus setting can be maintained throughout the sequence of exposures. The availability of low-cost computing power and the ready access to bundle adjustment and camera calibration programs (for example, via Internet) has meant that the techniques known as 'self-calibration' and 'on-the-job' calibration are now widely used.

4.4.1. On-the-Job Calibration

On-the-job calibration is the term which has been applied to a technique for determining the parameters of lens and camera calibration in-situ at the same time as the photography for the actual measurement of the object. The most likely scenario is that the object is not too large (say up to the size of a motor-car) and that a frame with pre-coordinated targets is placed over the object prior to photography. In this manner, the targeted frame and the object are exposed simultaneously so that control information is available on each exposure.

Most control frames occupy a space of a couple of cubic metres or less and an example is a cube made from light-weight aluminium bar which can be placed over a patient's head prior to close range photography for facial-cranial studies. The advantage of on-the-job calibration is most evident for non-metric cameras where there is a need to focus the lens for each epoch, and perhaps for each exposure. For a larger object, it may be inconvenient to completely enclose it within a targeted frame, so various researchers have devised "space-frames" based on interlocking rods and struts which can be simply assembled and placed in the field of view. Surveying tripods and levelling staves can serve as useful components of an on-the-job calibration network and can also be used for the provision of absolute control to the photography.

4.4.2. Self-Calibration

Self-calibration is an extension of the concept embodied in on-the-job calibration. Here, the observations of discrete targeted points on the object are used as the data required for both object point determination and for the determination of the parameters of camera calibration. The collinearity equations (see Chapter 2) are modified by the addition of the equations for lens distortion and suitable additional parameters and the resulting bundle of equations is solved simultaneously. If control targets with fixed coordinates are incorporated into the solution, then an absolute orientation can be derived for the camera’s location and orientation, and the coordinates of the targets on the object itself.

It is important to realise, although initially difficult to comprehend, that the self-calibration technique does not require any object-space control for the technique to be effective as a means of camera calibration. The geometrical arrangement of the camera stations, the intersection angles of rays from object points to cameras, the number of targeted points seen from a diversity of camera locations and the spread of targeted points across the image format are all important factors influencing both the precision of the coordinates of the targets on the object and the parameters of camera calibration.

As a simple example to explain the concept of self-calibration, consider the following scenario. A set of 50 targets, whose exact locations are unknown, are photographed from 8 camera stations, the location and orientation of which are also unknown. There are 3 unknowns per target point (X, Y, Z) and 6 per camera station, making a subtotal of 50 x 3 + 8 x 6 = 198 unknowns. If the focusing has been held fixed for the 8 exposures, the there are up to another 11 unknown parameters describing the principal distance, offsets of the principal point, radial and decentering distortion and the effects of shear in the sensor array and non-perpendicularity of that array to the principal axis to be determined. This brings the total number of unknowns to 198 + 11 = 209. Each observation of an imaged target produces 2 values (x and y), so that the total possible number of observations will be 8 images x 50 x 2 = 800 observations. As each observation relates to an observation equation in a combined solution, a not-inconsiderable redundancy of 800 - 209 = 591 exists for this solution.

Even when it is necessary to alter the camera focus for some exposure stations in a network, the method of self-calibration can be employed by treating those photographs as if they came from a different camera. The term block-variant is used in such cases. It is also possible to consider that the parameters for radial and decentering distortion have not altered even though the focus had been re-adjusted. In this case they could be termed block-invariant even though the principal distance would require different parameters for each change of focus.

To successfully recover the offsets of the principal point, it is useful to roll the camera through 90°, either between camera stations or at each camera station such that two exposures are captured at each view-point. Convergent photography is crucial for the successful recovery of the principal distance if the object under consideration is planar, and indeed it is also recommended for non-planar objects as the convergence enhances the strength of the geometric intersection of rays.

The self-calibration technique obviously has many advantages and is accepted as a ‘standard’ technique for the calibration of cameras. Obviously with digital cameras and high contrast retro-reflective targets which can be used in automatic processes to find target centroids, the technique can be fast and ‘hands-off’. However, there is a requirement for reasonable trial values for the all unknowns, and since a least squares iterative procedure is used for the solution of the self-calibration, there is a slight chance the solution may not find the optimum values for the unknowns if strong correlations exist amongst some of the unknowns or if the trial values are not close enough to ‘true’. Therefore it is sometimes recommended that a prior calibration technique, known as the analytical plumb-line calibration be undertaken before the self-calibration.

4.5 Analytical Plumb-Line Calibration

The parameters for radial and decentering distortion can be readily extracted by the analytical plumb-line technique. The principal distance and offsets of the principal point cannot be determined from this method. The simplicity of the analytical plumb-line calibration means that it is often used as either an independent check on self-calibration or as the means of obtaining trial values for the parameters of lens distortion prior to a bundle adjustment. The plumb-line technique provides a convenient method for determining the radial distortion at two different focus settings so that equation (2.6) can be invoked to determine its value at any other setting.

The plumb-line technique is based on the premise that, in the absence of distortion, a straight line in object-space will project as a straight line in image-space. The line formed on the image is measured, and departures from linearity are attributed to lens distortion. The technique was developed in the 1970s and has increased in popularity in recent years with the advent of automatic extraction of information from video images.

The formulation of the plumb-line technique relies on the fitting of straight lines to digitised sets of observed x,y coordinates on the image plane. Deviations of these lines from linearity are attributed to radial and decentering lens distortion (Figure 4.7). In other words, each digitised point can be thought of as consisting of a "true" position plus the effects of radial and decentering distortion. Sets of both approximately horizontal and approximately vertical lines need to be digitised to account for the effects of decentering distortion.

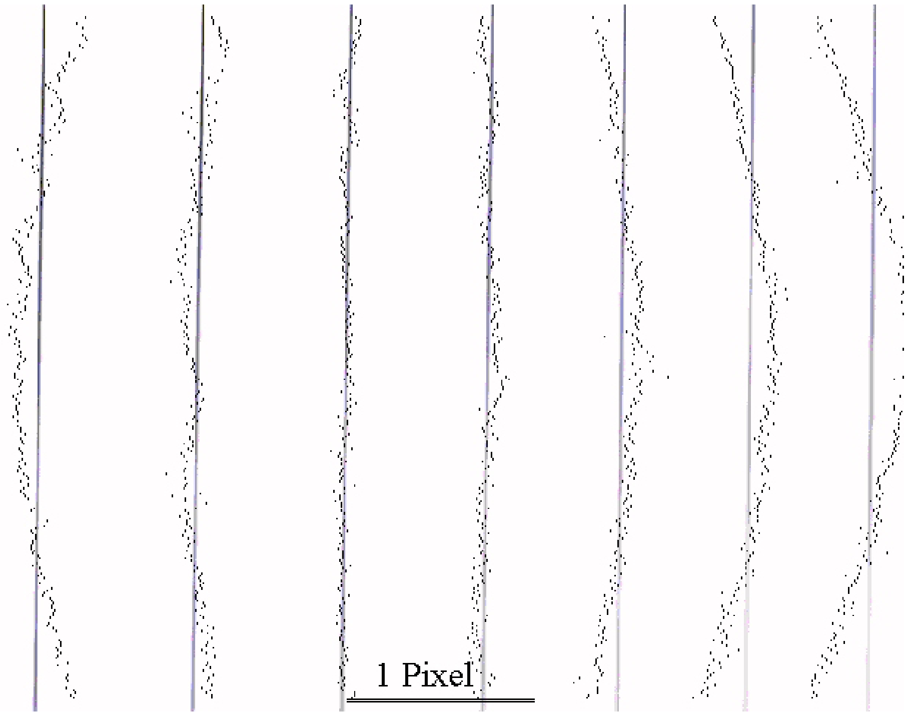

Figure 4.7. Before correction, best fit of plumb line data to a straight line

As an example to illustrate the efficiency of this technique, consider that 50 points were recorded in each of 10 horizontal and 10 vertical lines, then 1000 items of data would be available to describe those 20 lines plus the parameters x_p, y_p, K_1, K_2, K_3, P_1 and P_2. The extent of redundant information available is obvious and the strength of the solution is beyond reproach, except for the parameters x_p and y_p which can only be recovered with confidence in the case of fish-eye lenses where the distortion is very large. The results of applying the corrections to the image illustrated in Figure 4.8 are illustrated in Figure 4.9.

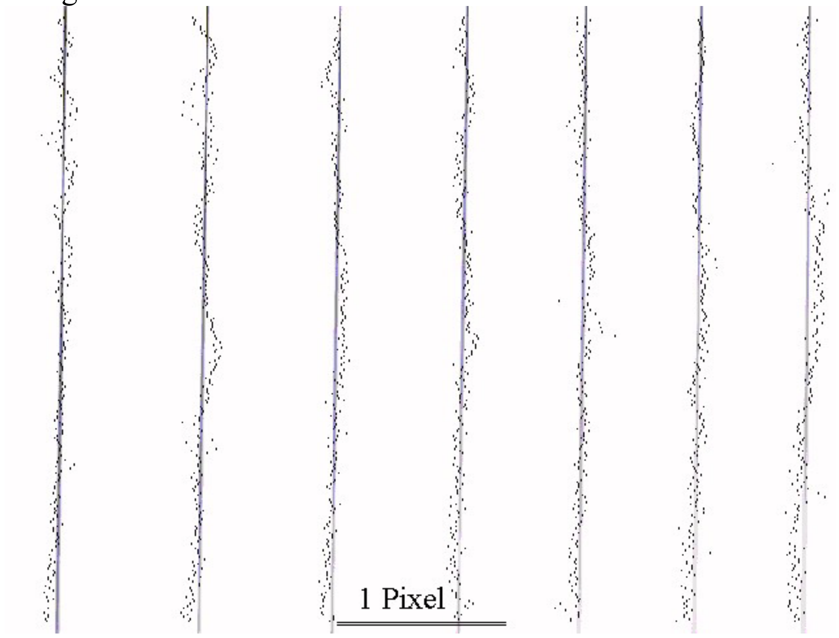

Figure 4.9 After correction, best fit of corrected data to a straight line

In general, the offsets for the principal point are not computed (equivalent to holding them fixed at zero for this phase of the total calibration). This is not a significant short-coming. The radial and decentering distortion values are now known with such a high degree of confidence that they can be entered into a self-calibration solution with strong *a priori* standard errors. This, in turn, has the effect of minimising the effects of correlation with the other parameters in the self-calibration solution.

The technique was initially seen as a laboratory calibration for close range cameras. The lines only have to be straight and not plumb, so that "wind-up" or "fold-away" calibration ranges can be quickly fabricated from string or fishing line using dark cloth as a contrasting back-drop). In recent years the technique has been extended to infinity focus situations for cameras ranging in size up to aerial and the targets used have included man-made objects such as the glass paneling on city buildings and even stretches of railway line.

One advantage of the plumb-line calibration technique has been demonstrated recently for video cameras operating in close-range situations. The simple C-mount lenses as fitted to most video cameras possess radial distortion that is characterised almost exclusively by the K_1, or cubic term. A frame, or a rectangle, with four straight lines surrounding the object is sufficient to determine the lens distortions present at each exposure. This could be classified as 'on-the-job plumb-line' calibration. The close-range nature of some tasks means that the magnitude of the radial lens distortion will vary significantly with slight re-focusing. The extension of the simple frame concept with four straight edges or engraved lines to robotics applications in hostile environments, such as nuclear power stations, and its application to situations where camera focusing may be automatic is worthy of consideration.

4.6 Summary

This chapter has provided a brief description of some methods of camera calibration. Some of those techniques might be better suited to a text book on the history of photogrammetry, but were included to illustrate the rapid advances in this area once computing power became commonplace. The concept of a dedicated test-range is one that most scientists and engineers appreciate, but the reality of digital cameras and image processing software is that test ranges are an unnecessary luxury.

The combination of a plumb-line calibration for an initial, but excellent, determination of radial and decentering distortion, followed by a 'self-calibration' procedure to firmly 'tie-down' all other parameters and keep unwanted correlations in check, is sometimes seen as the preferred option. The next chapter will provide case studies and examples of calibrations for specific work situations..

It is important for the practitioner not to become too concerned or involved with the mechanics of calibration techniques, since it is really the accurate determination of object coordinates which should be the primary aim. In routine tasks using the self-calibration technique, the photogammetrist should use the derivation of the calibration parameters only as a check to ensure that no unforeseen systematic error is present. The photogrammetrist only has to check that the additional parameters, including those of camera and lens calibration, are similar to previous adjustments to ensure a quality check on the entire adjustment procedure.

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